GUIDE TO THE EVALUATION AND EXPRESSION OF THE UNCERTAINTIES ASSOCIATED WITH THE RESULTS OF ELECTRICAL MEASUREMENTS.
AMENDMENTS ISSUED SINCE PUBLICATION

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Revision Note

This revision has been produced to update references and changes of title and address.

Some statements have been refined in the interests of accuracy and additional clarification inserted where deemed necessary.

Several mathematical statements have been amended.

Historical Record

DEF STAN 00-26/Issue 1 – published 15 February 1982.
The purpose of this Defence Standard is to provide all those associated with production, procurement and Service use of Defence equipment with guidance and advice on evaluating and expressing uncertainties associated with electrical measurements.

This Standard has been prepared under the aegis of the Defence Test Equipment Steering Committee Subcommittee A.

The principles and methods given in this Standard have been previously described in the following publications:

(a) ‘Guide to the expression of uncertainty of measurement in relation to the accuracy of instruments and specified test limits’ (MOD publication REMC/31/FR).

(b) ‘The expression of uncertainty in electrical measurements’ (NAMAS Document B 3003).

A major portion of the latter has been reproduced herein with the kind permission of the Department of Trade and Industry.

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This Standard has been agreed by the authorities concerned with its use and shall be incorporated whenever relevant in all future designs, contracts, orders etc and whenever practicable by amendment to those already in existence. If any difficulty arises which prevents application of the Defence Standard, the Directorate of Standardization shall be informed so that a remedy may be sought.

Any enquiries regarding this Standard in relation to an invitation to tender or a contract in which it is invoked are to be addressed to the responsible technical or supervising authority named in the invitation to tender or contract.
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Section One. General

0 Introduction

0.1 For engineering and Quality Assurance purposes it is important to be able to assess the significance of the figure obtained from a test or calibration, both in relation to specification limits, and to other measurements of the same attribute at different stages of procurement and in-Service use. For this, it is necessary that all measurements refer to National Standards through a traceable system, and that the figures be accompanied by an unambiguous statement of their probable accuracy. The latter is preferably given as an uncertainty figure which defines the limits within which the true value lies with a specified level of confidence.

1 Scope

1.1 This Defence Standard gives guidance on methods for evaluating the factors contributing to uncertainty in electrical measurements and for combining these into a single (±) limit figure.

1.2 Consideration is given to the economics of measurement uncertainty in the design, manufacture, calibration and testing of the product and to assessing the extent of the risk. It is recommended that the methods described herein should be adopted as standard procedure.

NOTE: This Defence Standard is concerned with the uncertainty of a single valued result of the measurement of a single parameter. It is not applicable, for example to the departure of a measured characteristic from a prescribed curve, neither is it concerned with sampling schemes for batch testing.
Section Two. Error and Uncertainty

2 Introduction

2.1 Quantitative statements on how nearly a product fulfils its specified function are obtained from measurements. In the Defence sector vitally important decisions may depend on such measurements and it is therefore essential that the results are fully and unambiguously yet concisely reported. All measurement involves the probability of error to some degree and often the magnitude of this error is by no means negligible when considering the operational performance of the product. This is particularly so in electrical measurements at radio and microwave frequencies.

2.2 The status of a measurement is quantified by the uncertainty figure, a term which recognizes that since the ‘true’ value of a quantity can never be known, neither ‘error’ nor ‘accuracy’ can be exactly determined. The word accuracy is best used only in a qualitative sense. Uncertainty is a statement of the limits of the range within which the true value is expected to lie in relation to the recorded result and it must include the probability of the true value lying within these limits. This probability is termed the ‘confidence level’, and is one minus the degree of risk that the true value may lie outside the uncertainty limits. In practice, confidence level is often expressed as a percentage figure, as opposed to the custom in mathematics of expressing probabilities with certainty as unity.

2.3 Thus, 100% confidence level means that the true value is guaranteed to lie within the uncertainty band, while 95% means that there is just a 1 in 20 chance of it lying outside the band. Although 100% confidence is an unattainable ideal limit, compared, say, with 99.7%, it has been quite common in the past to base the uncertainty estimate on the ‘worst case’, ‘guaranteed limits’ or ‘utmost confidence’ assessment of the several known possible sources of error. In this method these are assessed individually in terms of their maximum possible effect on the measured result and they are then added together arithmetically to obtain the overall uncertainty. This method obviously gives a high degree of confidence but with a pessimistic uncertainty figure, since it is extremely unlikely that several unconnected sources of error will all occur to their maximum extent in the same direction. This is illustrated by some examples to be given later. In most cases where the measurement uncertainty so calculated is still an order of magnitude less than the task demands (eg to meet specification limits) this may be acceptable, but in other cases it can lead to uneconomic or quite impracticable manufacturing specifications.

2.4 The recommended methods which are more realistic and informative, employ simple statistical principles to calculate the limits of uncertainty with a stated confidence level. They recognize that, in measurement, some small but known level of risk must be accepted if the task is to be practicable. It is impossible to make valid comparisons between the results of different measurements of nominally the same quantity without a knowledge of the confidence level which applies to the uncertainty limits.
2.5 In the methods described herein, two types of error (and associated uncertainty figures) are recognized, random and systematic. Subsequently, application of the theory of probability enables these two types to be combined to derive a single uncertainty figure with a stated confidence level.

Figure 1 illustrates graphically the error and uncertainty situation when making a simple measurement. Details in this figure will be mentioned under the headings which follow.

Figure 1  Illustration of measurement uncertainties
(This is simplified by assuming only one systematic uncertainty, that of the uncertainty of reference calibration of the measuring instrument used).

3 Random Error

3.1 When a measurement is repeated under the same conditions, the readings will be scattered over a range, provided the indicating system is sensitive enough to resolve the small differences involved. A classic example of such unpredictable fluctuations is provided by the effect of thermal noise in electrical measurements.
3.2.1 If a very large number of readings is taken and the results grouped into narrow adjacent bands of equal width over the whole range of values, the number of readings falling into each group, plotted vertically, will form a histogram, the envelope of which will approximate to the 'Normal' or Gaussian curve. This is the 'bell' shaped curve in Figure 1. The position of the peak of this curve approaches the arithmetic mean of all the results, as the number of measurements is increased indefinitely. For a small number of measurements, say 4 as shown in Figure 1, a different mean value will in general be obtained, and from a further set of 4 readings, yet another mean value. The difference between the mean value of a limited sample and the 'population mean' (the theoretical mean of an infinite number of measurements) is the random error of the mean of the sample. However, since the random error of a given sample size will differ on every occasion, the random uncertainty is a more relevant expression, giving the limits between which the random error will be, with a given probability of it being in this range.

3.3 The 'spread' of the repeated results governs the width of the Normal curve in Figure 1 and when suitably defined, provides a measure of the precision of the measurement. In the case of random uncertainty, this may depend on the method, the quality of the apparatus and in some cases on the skill of the person making the measurement. The statistic that expresses this property best is the root mean square of the deviations from the mean for the population, termed the standard deviation. It may be applied to the sample mean value or to a single measurement according to whether the mean of several readings is taken in every case, or whether only one reading is taken. Thus if $\sigma$ is the standard deviation for a single measurement, then $\sigma/\sqrt{n}$ is the standard deviation of the mean for a sample of $n$ measurements.

3.4 Figure 2 illustrates graphically how the standard deviation may be used, when the distribution is known to be Normal, to calculate the uncertainty for a given confidence level or vice versa.

3.5 The standard deviation of a restricted number of measurements can be used to determine bilateral limits about the mean value of the sample that embrace the population mean with a specified level of confidence.
3.5 (Contd)

For a Normal distribution, the percentage of values falling within limits equal to a factor (±) times the standard deviation, $\sigma$, is calculable. Referring to Figure 2, for a single measurement result there is a probability of 95% that it will fall in the range ±1.96 $\sigma$, and a risk of 5% that it will be in the shaded area. When there is a sample of n measurement results, these limits are reduced by a factor of $\sqrt{n}$ for the same confidence level. This illustrates the advantage of making more than one measurement.

3.6 In practice one has only an estimate of $\sigma$ based on a finite number of measurements, but allowance for the additional uncertainty involved can be made (see clause 7).

4 Systematic Error

4.1 Systematic error is unchanged when a measurement is repeated under the same conditions, but it becomes evident whenever the method is changed in a suitable way. Thus, before a systematic error can be determined and afterwards corrected, it must be identified; that is, related to some part of the measurement apparatus or procedure. Then, a modification of the method or the apparatus can be made that will reveal the error, so that a correction can be applied. Often, it is not possible to determine a systematic error precisely, or even at all. Then a systematic uncertainty is estimated, using as much experimental evidence as one can obtain.

4.2 One particularly simple systematic error and associated uncertainty arises from the calibration of the measuring apparatus or instrument. This total calibration uncertainty becomes a systematic uncertainty in subsequent use of the apparatus and if its probability density distribution is not stated, it is usually conservatively assumed to have a rectangular distribution as shown on the right of Figure 1. The main systematic error is corrected according to the calibration (Figure 1) but the systematic uncertainty remains to be combined with any other systematic uncertainty and the random uncertainty, as explained in section two.

5 Probability

5.1 Although certain concepts of the theory of probability are needed to express the characteristics of random and systematic uncertainties and the more advanced results of the theory are required for the combination of uncertainties, one can follow this Defence Standard without having a knowledge of these concepts and results. All the necessary conclusions are stated clearly.

5.2 The concept probability density distribution of continuously distributed probability does need special mention, however. In Figure 2 the probability $\Delta P$ that a continuously distributed result of a measurement has a value between $x$ and $x+\Delta x$ is expressed $\Delta P = f(x) \Delta x$ where $f(x)$ is the probability density distribution function. This value of $\Delta P$ is, for example, the area of the vertical strip of width $\Delta x$ and height $f(x)$ shown in Figure 2. The diagram should make this clear. Certainty is represented by unity and the function $f(x)$ is ‘normalized’ so that the area between the whole curve and the x-axis is unity.
This is the term used to denote the probability that the true value of a quantity is within ±U of the measured value, where U is termed the uncertainty in the measurement. As explained in the introduction, uncertainty and confidence level are both needed for a complete statement of a measurement result. If the probability density distribution is known, then uncertainty and confidence level are related if the standard deviation σ is given.

Thus, for a Normal distribution with standard deviation σ we have:

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<td>50.0%</td>
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<tr>
<td>1.96 σ</td>
<td>68.3%</td>
</tr>
<tr>
<td>2.0 σ</td>
<td>95.0%</td>
</tr>
<tr>
<td>2.58 σ</td>
<td>95.5%</td>
</tr>
<tr>
<td>3.0 σ</td>
<td>99.0%</td>
</tr>
<tr>
<td>3.0 σ</td>
<td>99.7%</td>
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The choice of confidence level for production testing is referred to in 15.
Section Three. Derivation of the Components of Uncertainty

7 Random Component of Uncertainty (\( U_r \))

7.1 For some measurements, for example in the low frequency and dc field, random uncertainty may not be very significant in relation to other contributions to uncertainty, but this cannot be assumed to be true for all electrical measurements. The first step should be to calculate the arithmetic mean or average of the results obtained.

7.2 The spread in the results, ie the range, reflects the merit of the measurement process and depends on the apparatus used, the method, and sometimes the person making the measurement. A more useful statistic, however, is the standard deviation of the sample \( s \). This is the root mean square difference from the arithmetic mean of the sample results. If there are \( n \) results for \( x \), where \( m=1,2 \ldots n \) and the sample mean is \( \bar{x} \) then

\[
\sigma = \sqrt{\frac{1}{n-1} \sum_{m=1}^{n} (x_m - \bar{x})^2} \tag{1}
\]

7.3 If further measurements are made, then, for each sample of results considered, different values for the arithmetic mean and standard deviation will be obtained. For large values of \( n \) these mean values approach a central limit value of a distribution of all possible values. This probability density distribution can usually be assumed, for practical purposes, to have the Gaussian or 'Normal' form.

7.4 From the results of a relatively small number of measurements an estimate, \( \sigma \) (est), can be made of the standard deviation of the whole population of possible values, of which the measured results are a sample, from the relation

\[
\sigma \text{(est)} = \sqrt{\frac{1}{n-1} \sum_{m=1}^{n} (x_m - \bar{x})^2} \tag{2}
\]

It will be noted that the only difference between \( \sigma \) (est) and \( s \) is in the factor \( 1/(n-1) \) in place of \( 1/n \), so that the difference becomes smaller as the number of measurements is increased.

7.5 This estimate of standard deviation can be used to calculate limits about the sample mean that will embrace the population mean with a specified probability. These limits are

\[
\bar{x} \pm U_r, \text{ where } U_r = t \left( \frac{\sigma \text{(est)}}{\sqrt{n}} \right) = \frac{ts}{\sqrt{(n-1)}} \tag{3}
\]
7.5 (Contd)

A value for ‘t’ (Student’s t factor) for the required confidence level and the number of measurements in the sample may be obtained from the table in annex C. It will be noted that this table shows that as the number of measurements in the sample is increased so t decreases asymptotically. The values for \( n = \infty \) are therefore the factors to use when the standard deviation for a near-normal distribution is known.

7.6 Equation 3 shows the benefit of increasing the number of measurements made in determining confidence in the mean. The benefit gets progressively less as the number is increased and it is usually not necessary to make more than about ten measurements. But for a measurement system and procedure that is going to be used frequently it is advantageous initially to make a large number of measurements on a component of high stability to obtain \( \sigma \) (est). Then, provided there are no changes in the system or procedure, only a relatively small number of measurements need be made on future occasions.

8 Systematic Component of Uncertainty (U_s)

8.1 Having determined the random (and measurable) component of uncertainty for the mean of the measurements, or determined that it is negligible, corrections to the mean value will be needed to establish traceability of the measurement units to national standards for those units. None of these corrections will, however, be known exactly, but all will have the characteristic that during the series of measurements that led to the measured mean value their values were constant. The residual and unknown amounts for which corrections cannot be made will be referred to as the contributions to the systematic component of uncertainty or systematic uncertainty.

8.2 The most obvious contribution to systematic uncertainty is that reported on the calibration certificate of the instrument from a laboratory placed higher in a national calibration system hierarchy and for which appropriately smaller values of uncertainty have been established. However, there can be, and usually are, other important contributions to systematic uncertainty in measurement that arise in the instrument user’s own laboratory. The successful identification and evaluation of these contributions is very dependent on a detailed knowledge of the measurement process and the experience of the person making the measurements. By planned variation of the conditions of measurement and the measurement process and averaging the results, identifiable systematic uncertainties can sometimes be corrected and thus prevented from contributing to the total uncertainty.

8.3 In determining systematic uncertainty it is necessary to consider in turn the measuring apparatus, the operational procedure, and the item under test. Although a truly comprehensive list of sources of systematic uncertainty cannot be given here, some contributions are fairly common in occurrence; in annex A some guidance is given for those that apply to electrical measurements. Whenever possible, corrections should be made for errors revealed by calibration or other considerations. On occasion, to simplify the measurement process it may be preferred to treat such an error, when it is small, as if it is a contribution to systematic uncertainty equal to (±) the uncorrected error magnitude.
8.4 Knowledge of the nature of the contributions will vary considerably. It may only be possible to assign limits to the uncertainty in a corrected mean value due to such a contribution. In this case it can be considered safe (in the sense of confidence in the result, that is, lack of risk) to assume that it is equally probable that, due to this contribution, the actual value is anywhere in the range of the limits; this is an assumption of a rectangular probability distribution.

8.5 When a number of distributions of whatever form are combined, the resultant distribution approximates to the Normal or Gaussian form. The degree of approximation depends upon the form, number and magnitudes of the individual components, but an important consequence is that the resulting overall uncertainty figure for systematic uncertainty has a somewhat higher confidence probability than if the distribution had been truly normal. For example, if there are a number of contributions with rectangular probability distributions and semi-ranges of $a_1$, $a_2$, etc, then the standard deviation of the resultant systematic uncertainty, $\sigma_s$, is

$$\sigma_s = \sqrt{\frac{a_1^2 + a_2^2 + \ldots a_n^2}{3}}$$

(4)

If $\sigma_s$ is then multiplied by a factor $K$ to place limits on the probability distribution, then provided $K$ is greater than about 1.5, the probability of values falling within the range $\pm K \sigma_s$ will always be greater than for a truly Normal distribution of the same standard deviation. Values of $K$ for specified probabilities are obtained from the last line of the table in annex C that is, the values of $t$ when $n = \infty$.

NOTE: The semi-range value is equal to half the difference between the maximum and minimum systematic contributions.

8.6 Accordingly, for the combination of contributions to systematic uncertainty, the procedure is first to determine the standard deviation for each actual or assumed form of probability distribution. If the standard deviations so obtained are $\sigma_{s1}$, $\sigma_{s2}$, etc, then the result of their combination for a specified minimum confidence level corresponding to the Normal distribution factor $K$ is

$$U_s = K \sqrt{\sigma_{s1}^2 + \sigma_{s2}^2 + \ldots \sigma_{sn}^2}$$

(5)

8.7 If one systematic component of uncertainty $a_d$, having a rectangular or 'U' shaped distribution is much larger than the others, calculation of the semi-range $U_s$, using (5) can result in a very pessimistic figure. The following test should be applied.
8.7 (Contd)

If, after calculation,

\[ U_s > \Sigma a_1, a_2, a_3, a_4, \ldots, a_n \]

(arithmetic summation of all the uncertainty components) ........... (6)

then \( U_s \) should be calculated from

\[ U_s = a_d + U_{sl} \] ............... (7)

Where \( U_{sl} \), calculated according to (5), is the resultant of all the other components and \( a_d \) is the dominant uncertainty component. This method nevertheless gives a "safe" ie somewhat pessimistic result.

9 Single Value of Total Uncertainty (U)

9.1 It will have been noted that the derivation of both the random and systematic components of uncertainty has been based on the characteristics of probability density distributions. The only significant difference is that the probability distribution for the random component has been derived from repeated measurements in the instrument user's own laboratory, whereas the probability distribution for the systematic component has been derived from a theoretical knowledge of contributing distributions, or conservative assumptions about the contributions from prior experimental work, such as that reported on the instrument calibration certificate. When both the main components of uncertainty have been derived for the same minimum confidence level the total uncertainty is given simply by

\[
U = \sqrt{U_r^2 + U_s^2} \] ............... (8)

If there is a dominant contribution to systematic uncertainty present (equations 6 and 7), then the total uncertainty can be expressed approximately as

\[
U = a_d + \sqrt{U_r^2 + U_{sl}^2} \] ............... (9)

10 Choice of Confidence Level (CL)

10.1 To permit valid and convenient comparisons of results it is clearly necessary that for a particular type of measurement the estimated confidence level on which uncertainty assessment is based should be the same at each level, or echelon, in a traceability chain. The choice of confidence level in terms of probability that a true value is within reported limits is, however, equally important. It can be readily shown
10.1 (Contd)

that when the instrument user’s own laboratory is separated by, say, two echelons from the national standards laboratory, the difference in the uncertainty for the instrument calibration between a confidence level of 95% ($1.96 \sigma$) and 99.7% ($3 \sigma$) can be most marked.

10.2 Another consideration is the maximum estimated confidence level on which uncertainty assessments should be based. Although maximum assurance in the total reported uncertainty for a measurement will always seem desirable, it is not realistic to attempt to achieve a confidence level greater than 99% ($2.58 \sigma$). Indeed, for some measurements where the additional uncertainties that are introduced at each echelon of measurement are relatively high, a lower level of confidence of 95% ($1.96 \sigma$) can be more appropriate.

10.3 As stated earlier, this Standard is intended to provide guidance in the expression of single values of uncertainty that include statements of estimated minimum confidence in the range of 95% to 99% probability. However, when the utmost confidence (approaching 100% probability) is required that the true value is within the stated uncertainty range, then it is recommended that the single value of uncertainty be derived as the arithmetic sum of $U_r$, calculated for a 99.7% confidence probability level (equation 3), and the estimated safe semi-range limits for he contributions to $U$. It must be appreciated that the result of deriving uncertainty by arithmetic summation in this way can lead to considerably increased ranges of uncertainty, with very small probabilities that the true values are near the range limits.

11 Expression of Measurements

The manner in which a measurement is to be expressed to indicate the level of confidence in the stated uncertainty should be, without abbreviation, as follows:

either

(a) $\bar{x} \pm U$ (estimated confidence level: $p\%$ min) where $p\%$ is the confidence in the limits for the corrected mean for a specified value in the range 95% to 99%;

or

(b) $\bar{x} \pm U$ (arithmetic summation of uncertainty contributions) when the utmost confidence in the limits is required at the place of measurement and achievable through the echelons of the traceability chain to national standards.

12 Procedures for the Determination of Measurement Uncertainty

12.1 Estimated confidence level in range 95% to 99%

12.1.1 Identify and list all corrections that have to be applied to measurements of a quantity.

12.1.2 Assign estimated semi-range limits, for which there is the utmost confidence, to the residual uncertainties in these corrections.
12.1.3 Assign a distribution form to each of the residual uncertainties and calculate the standard deviations (assume rectangular distributions if only limits can be estimated and refer to equation 4).

12.1.4 Choose the confidence level for the statement of measurement uncertainty and calculate the systematic uncertainty component \( U_s \), from equation 5, or equation 7, dependent on whether the criterion of equation 6 is met or not.

12.1.5 Assign a good estimate for the standard deviation based on the results of previous repeated measurements, or if this is not available make a minimum of four measurements to obtain an estimate from equation 2.

12.1.6 Repeat measurement to obtain a mean value for the quantity and apply all corrections (step 12.1.1). A mean value is of course available from (step 12.1.5) when the standard deviation of the measurement process has not been previously determined.

12.1.7 For the confidence level of step 17.1.4 calculate the random uncertainty component \( U_r \), from equation 3. The value of \( t \) is selected from the table in annex C for the number of measurements leading to the estimate of the standard deviation. The actual number of measurements that should be made will be determined by the relative importance of \( U_r \) to \( U_s \), in equations 8 or 9. A measurement should be repeated at least once however, to avoid operator mistake even when \( U_r \) is insignificant compared with \( U_s \), as it can be in certain electrical measurements.

12.1.8 Calculate a single value for total uncertainty from equations 8 or 9.

12.1.9 Report the result of the measurement process as the corrected mean value that has an uncertainty which is qualified with an estimate of confidence in the manner of clause 11.

12.2 Estimated utmost confidence level

12.2.1 As for steps 12.1.1, 12.1.2, 12.1.5 and 12.1.6.

12.2.2 Calculate the random uncertainty \( U_r \) as in step 12.1.7 for a confidence probability of 99.7%, or otherwise determine that it can be neglected.

12.2.3 Calculate a single value for total uncertainty as \( U_r \), plus the arithmetic sum of the semi-range limits of the contributions to systematic uncertainty, and report the measurement result as in clause 11.

12.3 Examples. Examples of the application of these step by step procedures to electrical measurements are given in annex B.
Section Four. Practical Considerations in Testing a Product

13 Uncertainty in Calibration of an Instrument

13.1 Instrument calibration must be regularly renewed, and checked whenever it is known or suspected that the instrument may have been subject to electrical overload, mechanical shock or an extreme environment. The uncertainty of the calibration must be specified by the calibration authority and the confidence level stated. In general practice a confidence level of 95% is used.

13.2 Additional uncertainty, not normally included in the calibration, arises from imperfect long-term stability of the instrument under the conditions in which it is used, and it is economic to use a high-quality instrument and to have it frequently calibrated until its stability is established.

14 Total Uncertainty in Testing

14.1 The uncertainties of the instrument and those of the measurement system in which it is used, should be combined as in equation 5. The distribution of the measurement results can usually be assumed to be Normal, with mean zero and a standard deviation according to equation 5. The measurement system and method will be planned to minimize as far as possible the uncertainty of the measurement result, eg using pads to reduce mismatch errors.

14.2 It is desirable whenever possible to obtain an estimate of total systematic error by measuring in the system a suitable device of proven stability which has been previously calibrated in a more precise system.

14.3 The standard deviation of the random component of uncertainty can be estimated from repeated measurements as described in clause 7.4 to 7.6. However, when many, nominally identical articles have to be tested in a nominally identical system over a period, it is advantageous to use accumulated data to obtain a reliable estimate of $\sigma$ (see equation 2). The value of $U_i$, the random component of uncertainty, is then obtained as $K \sigma$ where $K$ is obtained from the bottom line of the table of Student's 't' distribution in annex C.

If, however, the measured result is obtained by averaging several, say $N$ individual readings, the value of $U_i$ may be estimated as $K \sigma/\sqrt{N}$. Thus, if $N = 4$, taking 4 readings instead of 1, halves the value of $U_i$. In any particular case, the value of $N$ to be used will depend upon the time and the cost involved in taking the additional readings, the relative magnitudes of $U_i$ and $U_s$ and the cost of incorrect rejection of marginal products.

14.4 The total uncertainty is obtained using equation 8 or equation 9 as appropriate. We may then state, with the chosen level of confidence that the true value of the quantity differs from its measured value by less than $\pm U$. (It is of course necessary that the confidence levels for both systematic and random components of uncertainty be the same).
15 Uncertainty and Confidence Level in Testing to Limits

15.1 The total uncertainty, \( U \), of measurements made to test a product for conformance with Specification Limits, determines the margin(s) that must be allowed between Test Limits and the Specification Limits as will be seen in section five.

The confidence level for \( U \) affects the risk of making incorrect accept/reject decisions on the basis of the results of these measurements.

Two different risks are involved viz.

(a) Incorrect acceptance of ‘bad’ articles.

(b) Incorrect rejection of ‘good’ articles.

NOTE: The terms ‘good’ and ‘bad’, here and subsequently, refer to the true merit of the articles concerned ie if perfect measurements were made on them.

These two types of risk vary according to confidence level, (a) becoming less with increased confidence level whilst (b) increases - and vice versa.

It has been stated in section two that a confidence level of 95\% for electrical measurements generally represents a suitable and realistic compromise for most purposes and the arguments have even more force when testing to limits because of (a) – operational demands, and (b) – the ‘economic’ risk.

The implications of a confidence level of 95\% in production testing and the evaluation of the risks involved are discussed in section five.

15.2 It is important not to confuse higher confidence level with lower measurement uncertainty. The precision of a measurement is defined by \( \sigma \), the standard deviation of the combined random and systematic error distributions, and a lower figure can only be achieved by improving the measurement by some means.

\( U \) is obtained from \( K \sigma \) assuming a close approximation to a Normal distribution and the value of \( K \) depends upon the chosen confidence level.

Thus for confidence level = 95\%, \( K = 1.96 \)

= 99.7\%, \( K = 3.00 \)

(These and other figures can be obtained from the bottom line of the Student’s ‘t’ distribution at annex C.)

Therefore, setting a higher confidence level by using a higher value of \( K \), but with \( \sigma \) unchanged, increases \( U \) which calls for tighter Test Limits, with obvious consequences on production cost and output.

Specifying a confidence level greater than 99\% is probably unrealistic because the assumption of a Normal distribution for \( \sigma \) often breaks down near the ‘tails’ of the distribution and if, for some reason, it is essential to eliminate risk (a) altogether, the procedures for obtaining an ‘utmost confidence’ figure for \( U \) as described in Section Two should be used, or otherwise ‘factors of safety’ incorporated in the Specification.
Section Five. Effects of Digitizing Results

16 Effects

16.1 In an actual measurement there is always a minimum step $C$ in the expression of the result, where $C$ is the unit of the last digit in the numerical statement. Such a minimum step amounts to an uncertainty with a rectangular probability density distribution between $\pm \frac{1}{2}C$, which has a mean square value of $C^2/12$. The uncertainty describes what in other contexts has been termed ‘quantization error’.

16.2 When measurements are repeated to determine the random component of uncertainty, a finite $C$ results in some loss of information, but this loss is not considered significant if the range of the repeated results is greater than about $10C$. With this provision, if the set of results used to determine the square of the standard deviation or variance has the minimum step $C$, the value of variance obtained is, on average, $C^2/12$ larger than the value that would be obtained if $C$ were negligibly small. So the correction required to obtain the ‘true’ variance that would be obtained for a continuous random variable is $-C^2/12$. However, since the minimum step itself contributes $+C^2/12$ to the overall variance, the net effect is to require no correction on account of $C$ to equations 1 or 2 when calculating the standard deviation.

16.3 It is assumed that this result can be extended to the determination of random uncertainty, with a given confidence level. Then no correction to account for the minimum step is to be made to the random component of uncertainty, provided the minimum step $C$ used in the repeated measurements made to determine the uncertainty is the same as that used in the expression of the results.

16.4 If any arithmetic is used in processing the results of measurements, the number of significant figures used must be adequate to enable the answer to be expressed accurately to the same number of digits as were the original results of measurement.

16.5 It should be noted that the presence of a minimum step does not affect the method of calculating the mean value from a set of results of repeated measurements.
Section Six. Effect of Measurement Uncertainty in Production Testing

17 Introduction

17.1 In the production testing of articles supplied to a Performance Specification, measurement uncertainty, unless it is very small, will adversely affect the yield and the factory cost of the product. As a result it may influence the design, methods of manufacture and testing and quality control requirements.

17.2 This section discusses some of these implications and by means of a simplified but realistic model of the testing process it presents a practical procedure for estimating the additional cost and the risks resulting from this measurement uncertainty.

18 Specification Limits

18.1 The performance of a product can be specified in terms of limit values for the important parameters in three ways, viz.

(a) $T < B$ – the ‘one-way’ lower-limit case.

or

(b) $T < C$ – the ‘one-way’ upper-limit case.

or

(c) $F - L < T < F + L$ – the ‘two-way’ limit ± $L$, with ‘design-centre’ value $F$.

18.2 Here $T$ is the true value of the parameter concerned. In practice, of course, this can only be estimated from measurements, nevertheless to be generally meaningful Specification Limits, by convention, refer to true values.

19 Test Limits

19.1 The probability of error in the test measurement requires that Test Limits be ‘tighter’ than Specification Limits to reduce the otherwise considerable risk of shipping articles that are outside Specification.

19.2 The Test Limits should be set by arithmetically subtracting the uncertainty, $U$, from the Specification Limits.

19.3 $U$ should be derived by the methods laid down in section two.

19.4 Thus for the ‘two-way’ limit specification (case (c)), Test Limits will be $(F - L + U)$ and $(F + L - U)$.

19.5 For the ‘one-way’ limit specification (case (a) and (b)), a modified uncertainty figure $V$ – derived from $U$ – is used. This is explained in clause 5.9.

19.6 The Test Limits are thus $(B + V)$ and $(C - V)$ respectively.

20 The Effect of Confidence Level

20.1 As a result of setting the Test Limits, as stated in clause 19, the confidence level (CL) used in calculating $U$ now affects the risk of making incorrect verdicts when relating each measurement result to the Test Limits.
20.2 The recommended CL for calculating U is 95% (see Section Three) and this figure will be assumed in subsequent discussion. The justification for this, and the effects of higher or lower confidence levels will become apparent later.

20.3 The arguments and graphical data given below strictly apply only to large production runs, but nevertheless they are also useful where only small numbers are concerned.

21 Effect of Measurement Uncertainty

![Figure 3 Distributions for 'T' in Manufacture and Testing](image)

21.1 In figure 3, the abscissa represents values of T, i.e., true values of the parameter being measured, and shows the design value F at the centre with specified limits F+L and F-L. The ordinate represents Probability Density and is to an arbitrary scale.

NOTE: The superposition of these three distribution curves is purely for the purpose of illustration and should not be taken too literally.

21.2 U is shown much smaller than L. As will be seen later, it is unwise for U to exceed 0.3L otherwise the proportion of articles rejected becomes very large.
21.3 The wider curve represents the distribution of $T$ amongst the total production and this is assumed to be Normal with the mean value $= F$ and 'standard deviation' $D$. (This represents correct process control; $D$ being a measure of the 'spread' in manufacture estimated from a limited number of sample measurements.)

21.4 The two narrower curves represent the assumed normal distributions of $T$ for two individual articles with measured values $M_1 = F-(L-U)$ and $M_2 = F+(L-U)$, i.e. right on the Test Limits.

The shaded areas at the remote 'tails' of these two curves represent the risk of $T$ being outside the Specification Limits for these two particular articles (with measured values $M_1$ and $M_2$ respectively).

The areas under these two curves between $M_1$ and $(F-L)$ and between $M_2$ and $(F+L)$ represent the risk of incorrect rejection of articles with $M$ just outside the Test Limits, which although rejected may well be good articles. With CL = 95% there is a 19 in 20 chance of this.

Note that when $M = F+L$ or $F-L$ i.e. on the Specification Limit, there is still a 1 in 2 risk of incorrect rejection.

These proportions of incorrectly accepted or incorrectly rejected articles refer to individual articles with values near the test limits. When the whole production run is considered, with a normal distribution as shown in figure 3, the weighting of the relative probabilities of the various values of $T$ reduces the proportion of incorrectly rejected articles for given values of $U$ and $D$, and considerably reduces the proportion of incorrectly accepted articles. This can be seen intuitively from the curves in Figure 3.

21.5 Assessing the expected rejection rate. Looking now at the distribution (for $T$) of the total production in figure 3, the precision of manufacture is denoted by $D$, the 'standard deviation' of the distribution. The following facts can be elicited by inspection:

(a) The proportion of the total production subject to rejection depends upon the ratio $D/L$.

(b) The proportion of (a) likely to be rejected depends upon the ratio of $U/L$.

21.6 A quantitative evaluation can be made using the curves in figure 4. for discrete values of $D/L$, the percentage of the batch that can be expected to be rejected, $R$ is plotted against $U/L$.

21.7 The effect on $R$ of an increase in $U$ can readily be seen. Thus if $U = 0.2L$ then for $R = 5\%$, $D$ must be $0.4L$ approximately.

21.8 To reduce $R$ to $1\%$ $D$ must be reduced to $0.28L$.

21.9 If $U$ is $0.3L$ and $D$ is maintained at $0.4L$ $R$ will increase to about $11\%$. 

20
21.10 Hence it is recommended that U should not exceed 0.24L with $\leq 0.2L$ as a target figure for planning purposes.

21.11 It will be seen that there are three possibilities for reducing R:

(a) Relaxing the Specification by increasing L.

(b) Reducing D by improving process control.

(c) Reducing U by improving the measurement system.

21.12 Whether the expected rejection rate is tolerable or not, and if not, which of the above actions should be taken, depends on the technical feasibility and economics in each particular case. For example, it may well be economic to accept a high initial rejection rate and to reclaim the rejected articles by re-adjustment. This subject is outside the scope of the present Standard.

22 Incorrect Rejections

22.1 The considerable increase in R with increase in U has been noted. The proportion of correctly rejected articles for a particular value of D/L is given by the intercept with the ordinate, i.e. when U = 0. By subtracting this figure from R we can obtain the expected proportion of incorrectly rejected articles.

22.2 Thus for D/L = 0.4, U = 0.15L, R = 3.7%, the intercept at U = 0 is 1.3%, so the expected proportion of incorrect rejections is 3.7 - 1.3 = 2.4%.

23 Incorrect Acceptance

23.1 Nearly all the articles accepted are in fact correctly so. The expected proportion of incorrectly accepted articles for the example above is approximately 0.01%. This calculation assumes Normal distributions for the test measurements and production and may not be valid at the 'tails' of the distribution. Nevertheless the proportion is certainly very small which justifies the earlier statement that there is no good reason for choosing a confidence level greater than 95% in calculating U for test measurements.

24 Testing to a One-way Limit

24.1 When a single, upper or lower, limit is specified i.e. $T > B$ or $T < C$, the situation is different from the two-way limit case. The two one-way limit cases are mirror images of each other, so, in the interests of brevity, the argument will refer to the lower limit case.

24.2 The situation can be seen from figure 3, if the limit $T = (F-L)$ is called B and the upper limit, $T = (F+L)$ is disregarded. F is no longer applicable and the production mean, which we will call E, with standard deviation D, is no longer centred on a specified design figure.
24.3 With a Test Limit of $B + U$ the left-hand shaded area now represents the total risk of incorrect acceptance, all possible higher values of $T$ being acceptable. With $U$ calculated for a 95% confidence level the maximum risk is therefore 2.5%. As we have seen a 5% or 1 in 20 risk of incorrect acceptance is quite adequate for most purposes and this is most conveniently obtained by using $V$ for the purposes of setting the Test Limits instead of $U$, where $V = 0.84U$. (Assuming a close approximation to a Normal distribution for $U$.)

24.4 Thus the Test Limits become $B + V$ for the lower limit case and $C - V$ for the upper limit case.

The objective in manufacture will be to make $E$ as high as possible and $D$ as small as possible. Limitations will be imposed by technical feasibility and cost.

24.5 The proportion of rejections, $R$, that can be expected has been calculated and plotted in figure 5 in terms of $X$.

Where 

\[
X = \frac{E - (B + V)}{\sqrt{V^2 + 2.7D^2}}
\]

for the lower-limit case.

To reduce $R$ the value of $X$ must increase.

The corresponding expression for $X$ in the upper-limit case is

\[
X = \frac{(C - V) - E}{\sqrt{V^2 + 2.7D^2}}
\]

24.6 In practice it is most likely that the value of $E$ will be set by the design expectation, and the Specification Limits and Test Limits can be calculated for an acceptable figure of $R$ knowing the value for $D$ and $U$. The arguments are the same as for the two limit case and will not be further discussed here.

24.7 The economic significance of the uncertainty of the test measurements can readily be seen by inspection of these expressions.

24.8 It should be noted that $R$, the expected proportion of rejections, represents an average expectation based on a long production run with measurements made with the same system in the same way. The validity of the statistics in a practical case depends upon the truth of these assumptions and often these will not be very firmly based. Even so the estimates obtained are preferable to merely qualitative arguments.
Definitions:  
±L  - Specification Limits  
U  - Uncertainty of Test Measurements (CL = 95%)  
±(L-U)  - Test Limits  
D  - Standard Deviation of Manufacture  
R  - Expected Proportion of Articles Rejected
Acceptance Criterion \( M > B + V \)

\[ X = \frac{E - (B + V)}{\sqrt{V^2 + 2.7D^2}} \]

Where:

- \( V = 0.84U \) (Uncertainty of Test measurement)
- \( C.L = 95\% \)
- \( E = \) Mean value of \( T \) for Manufactured Articles
- \( D = \) Std Dev \( " " " " " \)
- \( R = \) Expected proportion of articles rejected

Figure 5 Expected Proportion of Articles Rejected When Testing to One-way Limit
Some Sources of Systematic Uncertainty in Electrical Measurement

A.1 Instrument Calibration

A.1.1 The uncertainties assigned to the values on a certificate for the calibration of an instrument are certified as being correct at the time and under the conditions of calibration. The values are used to correct the instrument indications and/or recorded values.

A.2 Temporal Stability

A.2.1 The performance of all instruments, whether measuring equipment or reference standards, must be expected to drift slowly with time. Thus the instrument user in seeking the most probable values at the time of his measurements has to assess the drift since the last calibration. This assessment will need to be based on the results of previous calibration. It is helpful to display the accumulated data in graphical form for this purpose. The corrections that are applied for drift in values are subject to uncertainties that are systematic in their effect on the measurement results. The magnitude of the drift in values will of course determine the required periodicity of calibration.

A.3 Value of a Quantity and Electromagnetic Frequency of Measurement

A.3.1 In considering a calibration for an instrument there are always practical and economic factors which limit the number of calibration points that can be provided. Consequently, it becomes probable that the quantity to be measured and its frequency, if it is an rf measurement, will be different from the values for which a calibration has been provided.

A.3.2 When the quantity is between two calibration values, then consideration needs to be given to any systematic uncertainty arising from, for example, scale non-linearity, and range switching.

A.3.3 If the measurement frequency falls between two calibration frequencies, it will also be necessary to assess the additional interpolation uncertainty that this can introduce. As high rf and microwave frequencies, one can only proceed with a degree of confidence if:

(a) a theory of the instrument is available to predict a characteristic for the interpolation between the calibration frequencies, or there is accumulated evidence of the performance of many other models of the same instrument; and

(b) the performance of the actual instrument being used has been explored with a swept frequency measurement system to verify the absence of effects due to manufacturing imperfections.

A.4 Interconnection of Apparatus

A.4.1 The manner in which an instrument is connected to other items of apparatus and the characteristics of these items can significantly affect the performance of an instrument. Thus, unless the method of use of an instrument is identical to that used during calibration, additional contributions to systematic uncertainty in the results obtained are likely.
A.4.2 The possible causes of such uncertainty are legion and their successful identification and evaluation can prove difficult. The classic approach under such circumstances is a planned variation of the measurement conditions to assist detection.

A.4.3 In reactance measurements at low frequency, for example, the results can be affected by incompatibility of terminal arrangements (e.g., connection from stud terminals to a ‘sexless’ coaxial connector), lead capacitance and inductance, earthing and screening arrangements.

A.4.4 In power and attenuation measurements of rf and microwave frequencies, it is necessary to consider the complex voltage reflection coefficients at the transmission line terminal planes of the instrument. It is usually only practical to determine the modulus of such a reflection coefficient and there is, therefore, uncertainty about the relative phase angle of the mismatch of the instrument to that of its source and, in the case of attenuation measurements, its load. The nature of this uncertainty differs from some others that have been mentioned in that theory indicates the form of its probability distribution. For example, in the transfer of power between a source and a load of voltage reflection coefficients \( \rho_G \) and \( \rho_L \) respectively the fraction of the available power transferred to the load is

\[
\frac{(1 - |\rho_G|^2)(1 - |\rho_L|^2)}{1 - |\rho_G\rho_L|^2}
\]

A.4.5 The uncertainty is in the complex quantities \( \rho_G \) and \( \rho_L \), in the denominator and can be expressed as ‘a cos \( \phi \)’ where ‘a’ equals \( 2|\rho_G|\rho_L \) and ‘\( \phi \)’ is the relative phase angle assumed to have any value with equal probability in the range 0 to \( \pi \). As the uncertainty magnitude is in terms of \( \cos \phi \), the probability distribution is U-shaped and has a standard deviation of \( a/\sqrt{2} \).

A.5 Measurement (or Service) Conditions

A.5.1 If the laboratory measurement environment is different from that for a calibration, then due allowance has to be made for any influence conditions such as temperature, relative humidity, presence of objects in the measurement electromagnetic field. Also it is necessary to be aware of the possible effect of operating electrical conditions, such as power dissipation and applied voltage, differing from those for a calibration. Examples of the importance of such contributions to systematic uncertainty, although more common in dc and lf measurements, do also occur in rf measurements.
Examples of Application to Electrical Measurement

B.1 Measurement of a 10 kΩ Resistor by Potentiometric Intercomparison

B.1.1 Contributions to Systematic Uncertainty (Step 12.1.2).

Semi-range limits (ppm)

Reference resistor, $R_s$ (nominal 10 kΩ)

Calibration 2
Stability (between calibrations) 4
Service conditions (temperature) 1.4

Potential differences

$V_s$ across $R_s$ 0.5
$V_x$ across $R_x$ 0.5

Arithmetic sum 8.4

B.1.2 Systematic Uncertainty (Steps 12.1.3, 12.1.4)

Assume rectangular distributions and calculate standard deviation of combination from equation 4.

$\sigma_s = 2.7$ ppm

From equation 5

$U_s$ (est CL : 99% min) = 7.0 ppm
$U_s$ (est CL : 95% min) = 5.3 ppm

B.1.3 Random Uncertainty (Steps 12.1.5, 12.1.6, 12.1.7)

Repeatability of Ratio $V_x/V_s$ (major component: power supply stability).

Departure from unity in ppm (all corrections applied).

$+ 10.0, + 11.0, + 10.5, + 11.5$

Mean value = 10.8 ppm.

From equation 2, the estimated standard deviation = 0.65 ppm. The random uncertainty, $U_r$, is calculated from equation 3 for values of Student’s $t$ for the required confidence levels:

$U_r$ (est CL : 99.7%) = 3.0 ppm
$U_r$ (est CL : 99.0%) = 1.9 ppm
$U_r$ (est CL : 95.0%) = 1.0 ppm.
**ANNEX B (Continued)**

**B.1.4 Total Uncertainty (Step 12.1.8)**

From equation 8

\[ U \text{ (est CL : 99\% min)} = 7.3 \text{ ppm} \]
\[ U \text{ (Est CL : 95\% min)} = 5.4 \text{ ppm} \]

By arithmetic summation (taking 99.7\% est CL for \( U_r \))

\[ U = 11.4 \text{ ppm} \]

**B.1.5 Alternative Expression of Results**

Measured value

\[ 10 \, 000.11 \, \Omega \pm 0.07 \, \Omega \text{ (estimated confidence level: 99\% min)} \]

or

\[ 10 \, 000.11 \, \Omega \pm 0.05 \, \Omega \text{ (estimated confidence level: 95\% min)} \]

or

\[ 10 \, 000.11 \, \Omega \pm 0.11 \, \Omega \text{ (arithmetic summation of uncertainty contributions)} \]

**B.1.6 Comment.** As mentioned in 10.3, it should be appreciated, in considering the alternative expressions given above, that the effect of choice of confidence level on the value reported for total uncertainty will be greatly increased when the same processes of calculation are followed at a number of echelons in a traceability chain.

Also it will be noted in this example that the random component of uncertainty is hardly significant in its effect on the total uncertainty \( U \) for confidence levels up to 99\%, and that such prior knowledge would simplify the measurement process.

**B.2 Calibration of a Coaxial Power Meter at 10 mW and 9 GHz by Direct Substitution for a Working Standard Power Meter**

**B.2.1 Contribution to systematic uncertainty (step 12.1.2)**

<table>
<thead>
<tr>
<th>Semi-range limits (%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Calibration</td>
<td>0.8</td>
</tr>
<tr>
<td>Stability (between calibrations)</td>
<td>0.5</td>
</tr>
<tr>
<td>Frequency interpolation</td>
<td>0.2</td>
</tr>
<tr>
<td>Mismatch</td>
<td></td>
</tr>
<tr>
<td>Effective source vswr 1.06 (( \rho = 0.029 ))</td>
<td></td>
</tr>
<tr>
<td>Power meter vswr 1.10 (( \rho = 0.048 ))</td>
<td></td>
</tr>
<tr>
<td>From annex A, uncertainty limits</td>
<td>0.28</td>
</tr>
<tr>
<td>Unknown power meter</td>
<td></td>
</tr>
<tr>
<td>Mismatch</td>
<td></td>
</tr>
<tr>
<td>Effective source vswr 1.06 (( \rho = 0.029 ))</td>
<td></td>
</tr>
<tr>
<td>Power meter vswr 1.34 (( \rho = 0.145 ))</td>
<td></td>
</tr>
<tr>
<td>From annex A, uncertainty limits</td>
<td>0.8</td>
</tr>
<tr>
<td>Arithmetic Sum</td>
<td>2.58</td>
</tr>
</tbody>
</table>
**B.2.2 Systematic uncertainty (step 12.1.3, 12.1.4)**

If the Calibration of the standard has a reported confidence level of 95% (1.96 x standard deviation)

\[ \sigma_{s_1} = 0.408\% \quad (0.8/1.96) \]

Assume rectangular distribution for the stability and frequency interpolation uncertainty contributions of the standard.

From equation 4

\[ \sigma_{s_2} = 0.311\% \]

For the mismatch contribution, the probability distributions are U-shaped with a standard deviation equal to \( a/\sqrt{2} \) (annex A)

\[ \sigma_{s_3} = 0.599\% \]

From equation 5

\[ U_r(\text{est CL: }95\% \text{ min}) = 1.55\% \]

**B.2.3 Random uncertainty (step 12.1.5, 12.1.6, 12.1.7)**

Repeatability of readings in mW of standard power meter (all corrections applied) for a reading on the unknown meter of 10.00 mW were:

10.04; 10.07; 10.03; 10.06

Mean value = 10.05 mW

From equation 2, the estimated standard deviation = 0.18%. The random uncertainty, \( U_r \), is calculated from equation 3 for values of Student’s \( t \) for the required confidence levels:

\[ U_r(\text{est CL : } 99.7\%) = 0.83\% \]
\[ U_r(\text{est CL : } 95.0\%) = 0.29\% \]

**B.2.4 Total Uncertainty (Step 12.1.8)**

\[ U(\text{est CL : }95\% \text{ min}) = 1.57\% \]

By arithmetic summation (taking 99.7% est CL for \( U_r \))

\[ U = 3.41\% \]

**B.2.5 Recommended Expression of Results**

The incident rf power for a power meter reading of 10.00mW is

10.05mW +/- 1.57% (estimated CL: 95% min)
**B.3** The Measurement of the Output Power from a Wide Band rf Amplifier Using a Production Line Automated Test Equipment (ATE) Calibrated Against a Reference Standard Power meter

**B.3.1** Contributions to Systematic Uncertainty (Step 12.1.2)

<table>
<thead>
<tr>
<th>Contribution</th>
<th>Semi-range limits (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference Standard Power meter (RS)</td>
<td></td>
</tr>
<tr>
<td>Calibration</td>
<td>2</td>
</tr>
<tr>
<td>Stability (between calibrations)</td>
<td>1</td>
</tr>
<tr>
<td>Frequency interpolation</td>
<td>1</td>
</tr>
<tr>
<td>Power resolution of ATE</td>
<td>2.0</td>
</tr>
<tr>
<td>Frequency interpolation of ATE</td>
<td>1.4</td>
</tr>
<tr>
<td>Stability of ATE</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Mismatch between RS and ATE input
Effective Generator VSWR 1.2 ($\rho = 0.091$)
ATE input VSWR 1.3 ($\rho = 0.13$)

From annex A, uncertainty limits 2.4

Mismatch between ATE and RF Amplifier
RF Amplifier effective output VSWR 2.4 ($\rho = 0.41$)
ATE, input VSWR 1.3 ($\rho = 0.13$)

From annex A, uncertainty limits 11

Arithmetic sum 22.8

**B.3.2** Systematic Uncertainty (Steps 12.1.3 and 12.1.4)

In the list of contributions under **B.3.1** above, it is seen that the mismatch error of semi-range 11% has a much wider range than any other contribution, and it remains to determine whether this satisfies equation 6 of clause 8.7 or not. To find the $U_{s1}$ referred to in 8.7, we assume rectangular distributions for the non-mismatch contributions, and a U-shaped distribution for the first mismatch contribution for which the standard deviation equals $a/\sqrt{2}$ (annex A) ie $2.4/\sqrt{2}$

Then, with equation 4 for first six contributions,

$$\sigma_{s1} = \left(16 + \frac{5.76}{2}\right)^{1/2} = 2.86\% \text{ and } U_{s1} = 1.96 \times 2.86 = 5.61\%$$

Now in equation 6, $2.2 \times 5.6 = 12.3$ is not less than 11. So we do not use equation 7 to find $U_1$, but from equation 5 we have

$$U_1 = 1.96 \left[\left(\frac{11}{\sqrt{2}}\right)^2 + (2.86)^2\right]^{1/2} = 16.24\%$$

So $U_1$ (est CL: 95% min) = 16.24%
**B.3.3** Random Uncertainty (Steps 12.1.5, 12.1.6, 12.1.7)

Repeatability of readings in W of output power including stability of RF Amplifier:

120; 121; 120; 119; 122

Mean value = 120.4 W

From equation 2 the estimated standard deviation = 1.14 W. The random uncertainty, $U_r$, is calculated from equation 3 for values of Student’s $t$ for the required confidence level.

$$U_r (\text{est CL: 95\%}) = 1.18\% (1.42 \text{ W})$$

**B.3.4** Total Uncertainty (Step 12.1.8)

$U$ (est CL : 95\%) = 16.29\% (19.61 W)

By arithmetic summation (taking 99.7\% est CL for $U_r$)

$U = 25.7\% (31.0 \text{ W})$

**B.3.5** Recommended Expression of Results.

The output RF power of the amplifier is

120.4 W ± 19.6 W (estimated confidence level: 95\% min).
### Student’s t Distribution

#### C.1 Values of t for specified confidence probability P as a function of the number of measurements, n.

<table>
<thead>
<tr>
<th>n</th>
<th>0.50</th>
<th>0.683</th>
<th>0.950</th>
<th>0.955</th>
<th>0.990</th>
<th>0.997</th>
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<td>1.84</td>
<td>12.7</td>
<td>14.0</td>
<td>63.7</td>
<td>236</td>
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<td>2.87</td>
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</table>

| n  | 0.675 | 1.00  | 1.96  | 2.00  | 2.58  | 3.00  |
Symbols

D.1 The meanings of symbols of measurement have been given in the text where they occur, but are repeated here for later convenience of reference:

\( x \)

- a value among \( n \) measurements where \( m = 1,2...n \)

\( \bar{x} \)

- arithmetic mean value of a sample of \( n \) measurements

\( s \)

- standard deviation of a sample of \( n \) measurements

\( \sigma \)

- standard deviation for the distribution of a random variable

\( \sigma \) (est)

- an estimate of the standard deviation of the distribution of a whole population (infinite number) of measurements based on a sample of \( n \) measurements:

\[
\sigma \text{(est)} = s \sqrt[\frac{n}{n-1}}
\]

\( K \)

- factor for tolerance limits with associated probability for a normal distribution

\( t \)

- Student’s \( t \) variate for the difference between sample mean, \( \bar{s} \), and population mean, \( u \) for a specified confidence probability:

\[
|t| = \left| \frac{\bar{x} - u}{\sigma \text{(est)}} \right| \sqrt{n}
\]

\( U_r \)

- value of uncertainty in a measurement result due to random (and measurable) effects for a specified confidence probability

\( \sigma_s \)

- standard deviation for a systematic contribution to uncertainty in measurement having an assumed or given distribution of possible values

\( a \)

- semi-range of systematic contribution to uncertainty

\( U_s \)

- value of uncertainty in a measurement result due to systematic effects for a specified confidence probability

\( U \)

- single value of uncertainty in a measurement result obtained from combining \( U_r \) and \( U_s \) for a specified confidence probability.
The following Defence Standard file reference relates to the work on this Standard – D/D Stan/357/02/00.

Contract Requirements

When Defence Standards are incorporated into contracts users are responsible for their correct application and for complying with contract requirements.

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